SPHERICAL TRIGONOMETRY

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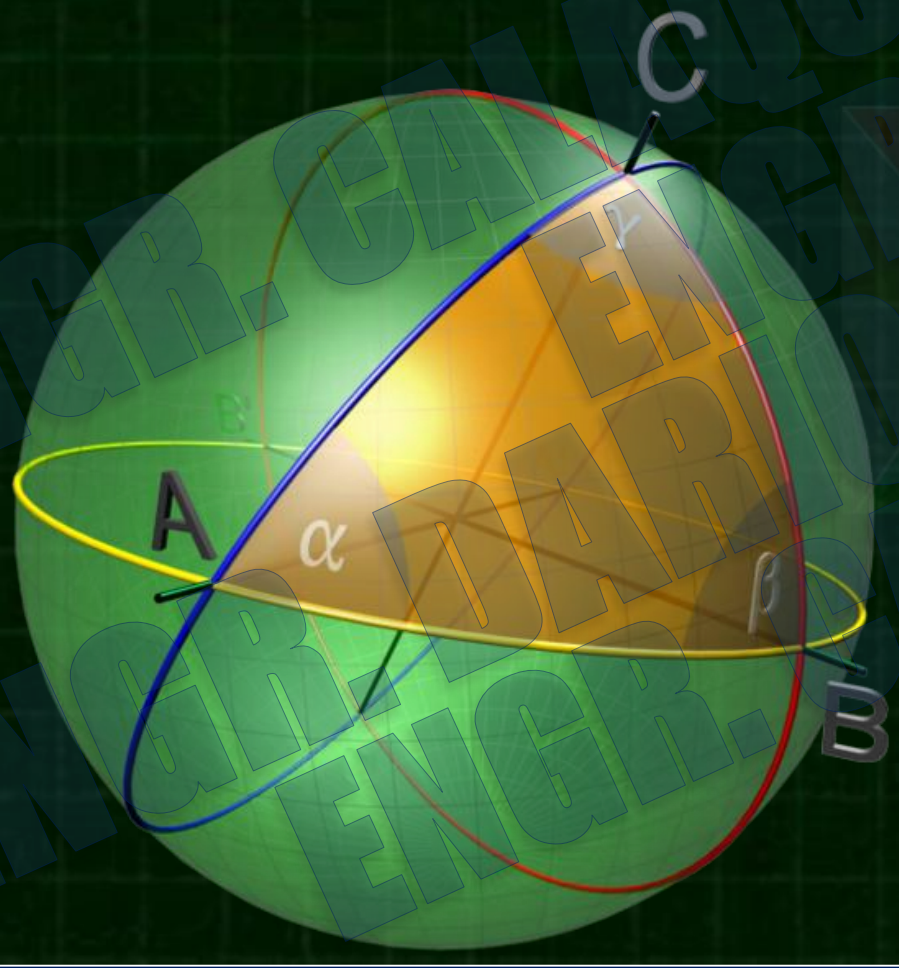
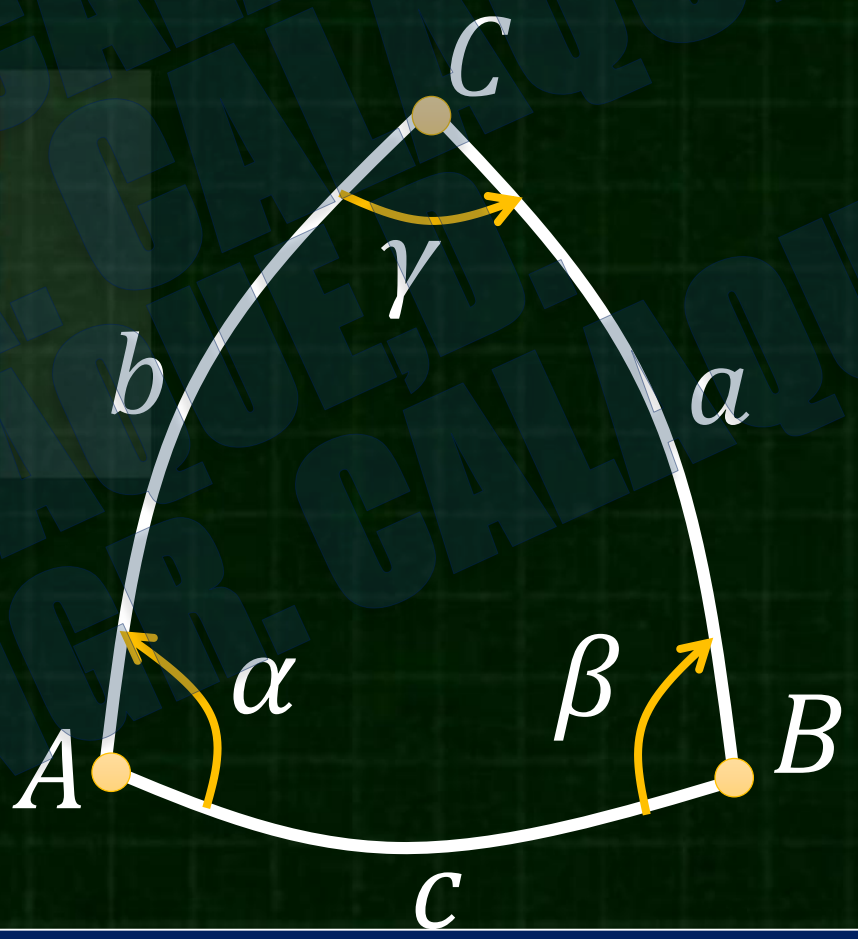
SPHERICAL TRIANGLE





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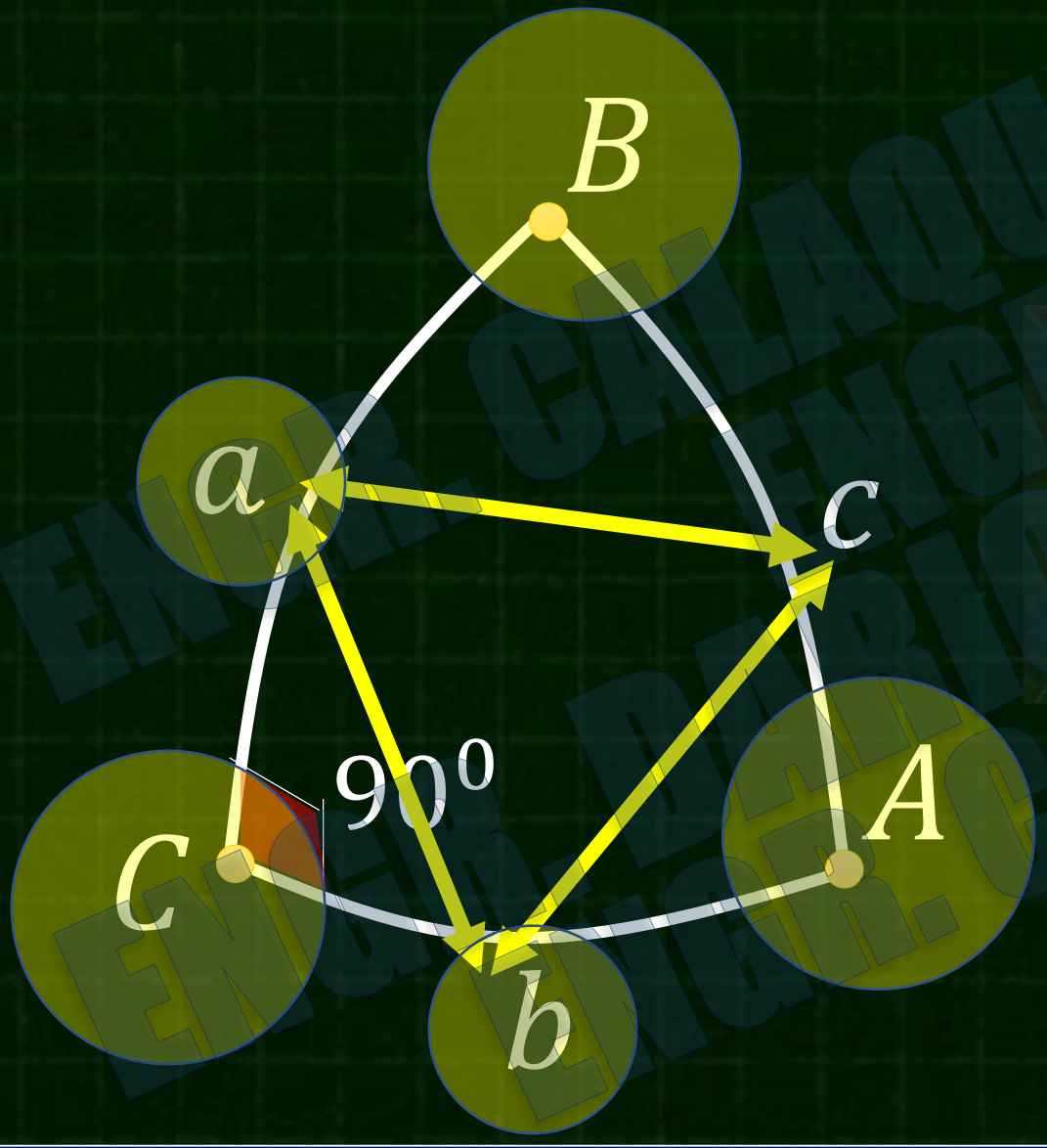
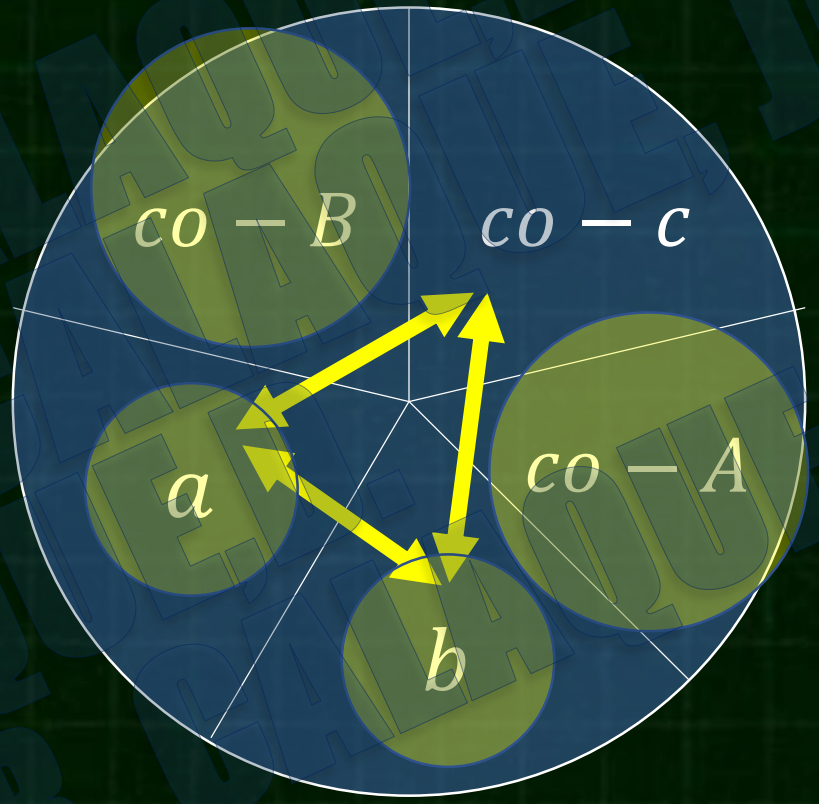
RIGHT SPHERICAL TRIANGLE

USE JOHN NAPIER'S CIRCLE





Rule 1: TAN – AD – RULE

Rule 2: CO – OP – RULE

Angle C is assumed to be 90^0

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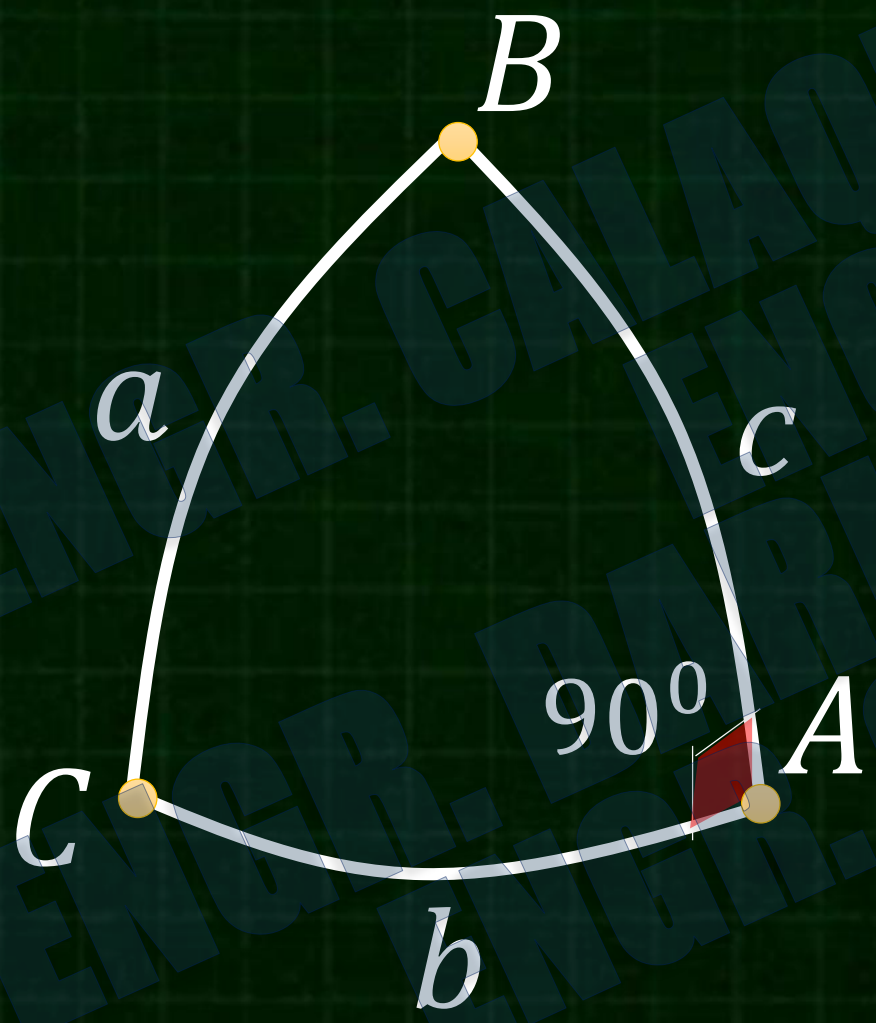
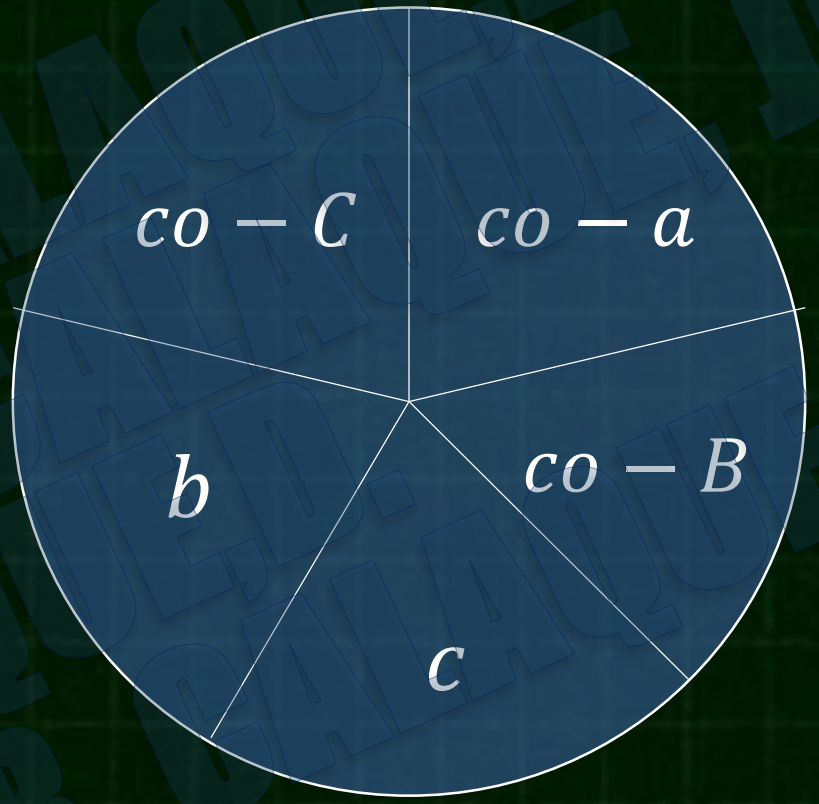
RIGHT SPHERICAL TRIANGLE

USE JOHN NAPIER'S CIRCLE





Rule 1: TAN − AD − RULE

Rule 2: CO − OP − RULE

Angle A is assumed to be 90^0

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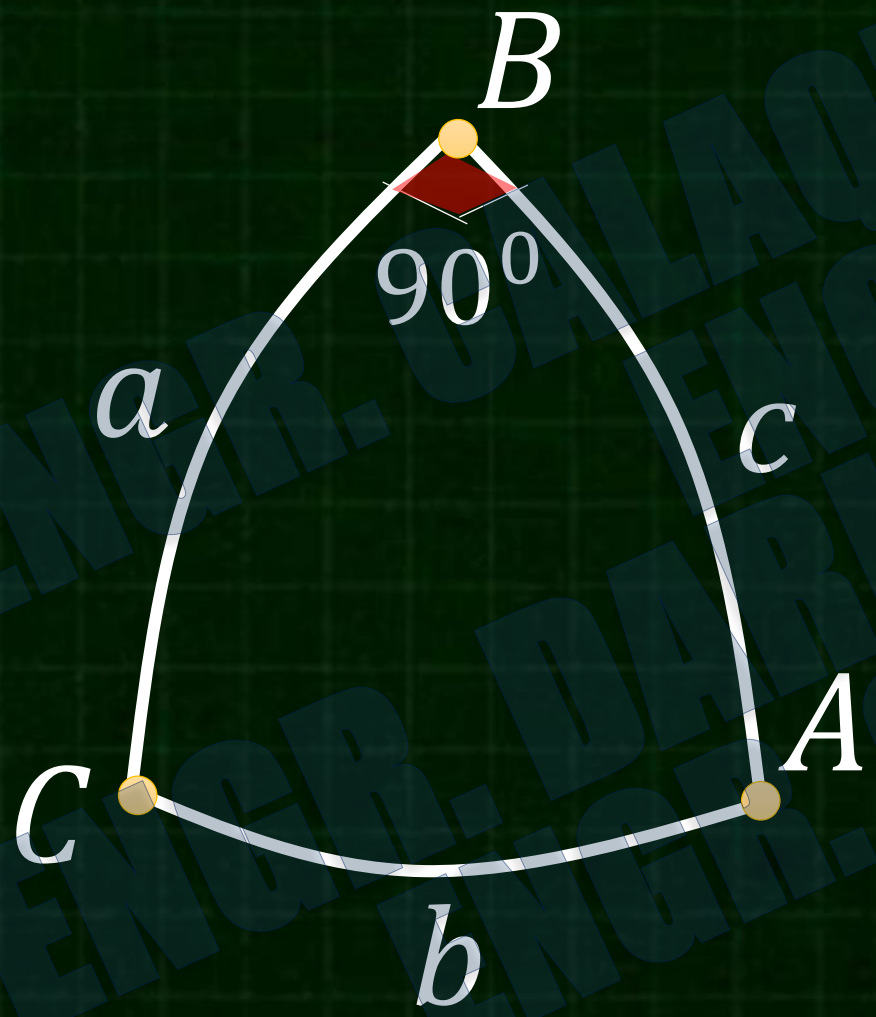
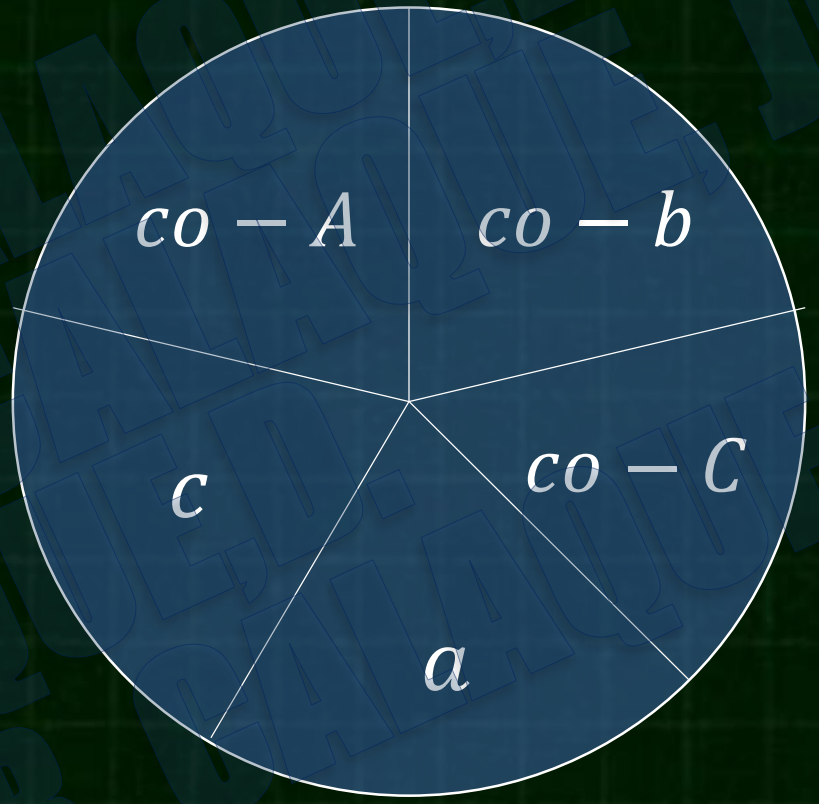
RIGHT SPHERICAL TRIANGLE

USE JOHN NAPIER'S CIRCLE





Rule 1: TAN − AD − RULE

Rule 2: CO − OP − RULE

Angle B is assumed to be 90^0

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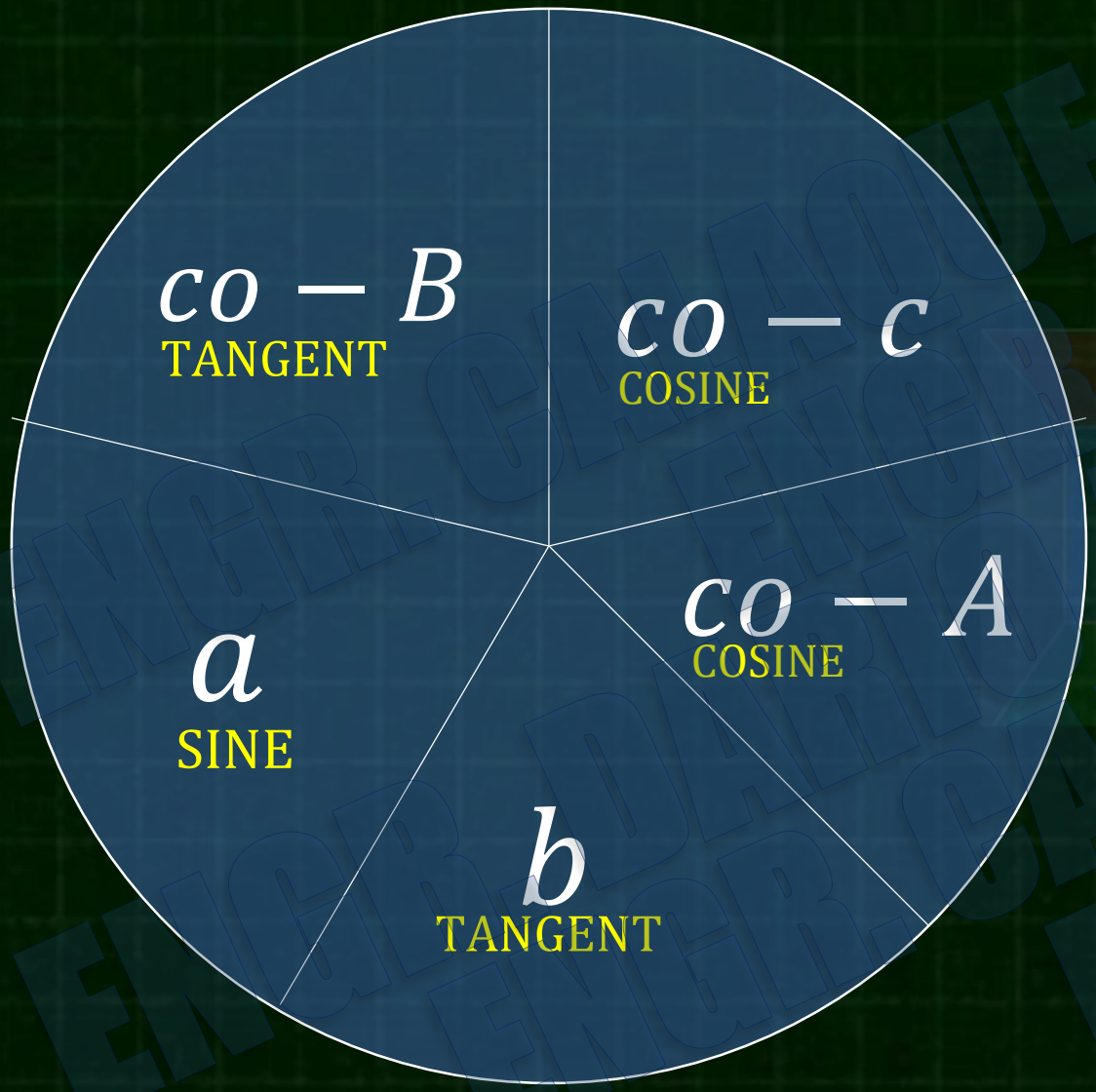
NAPIER'S CIRCLE



Rule 1: TAN – AD – RULE

The sine of any middle part is equal to the product of the tangent of the adjacent parts

$$\sin a = \tan(co - B)\tan(b)$$

$$\sin a = \cot B \tan(b)$$

Rule 2: CO – OP – RULE

The sine of any middle part is equal to the product of the cosine of the opposite parts

$$\sin a = \cos(co - c)\cos(co - A)$$

$$\sin a = \sin c \sin(A)$$

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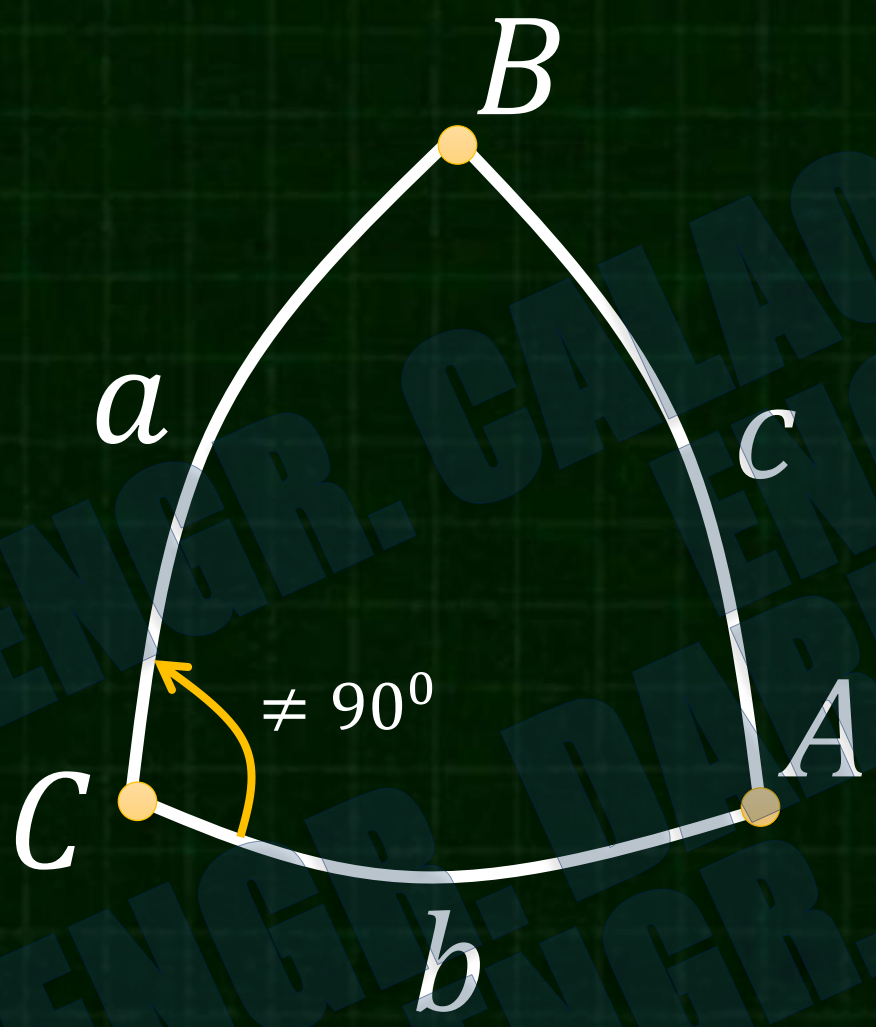
OBLIQUE SPHERICAL TRIANGLE

USE SPHERICAL TRIGONOMETRY LAWS



SINE LAW:

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

COSINE LAW (FOR THE SIDES):

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$
$$\cos b = \cos a \cos c + \sin a \sin c \cos B$$
$$\cos c = \cos a \cos b + \sin a \sin b \cos C$$

COSINE LAW (FOR THE ANGLES):

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a$$
$$\cos B = -\cos A \cos C + \sin A \sin C \cos b$$
$$\cos C = -\cos A \cos B + \sin A \sin B \cos c$$

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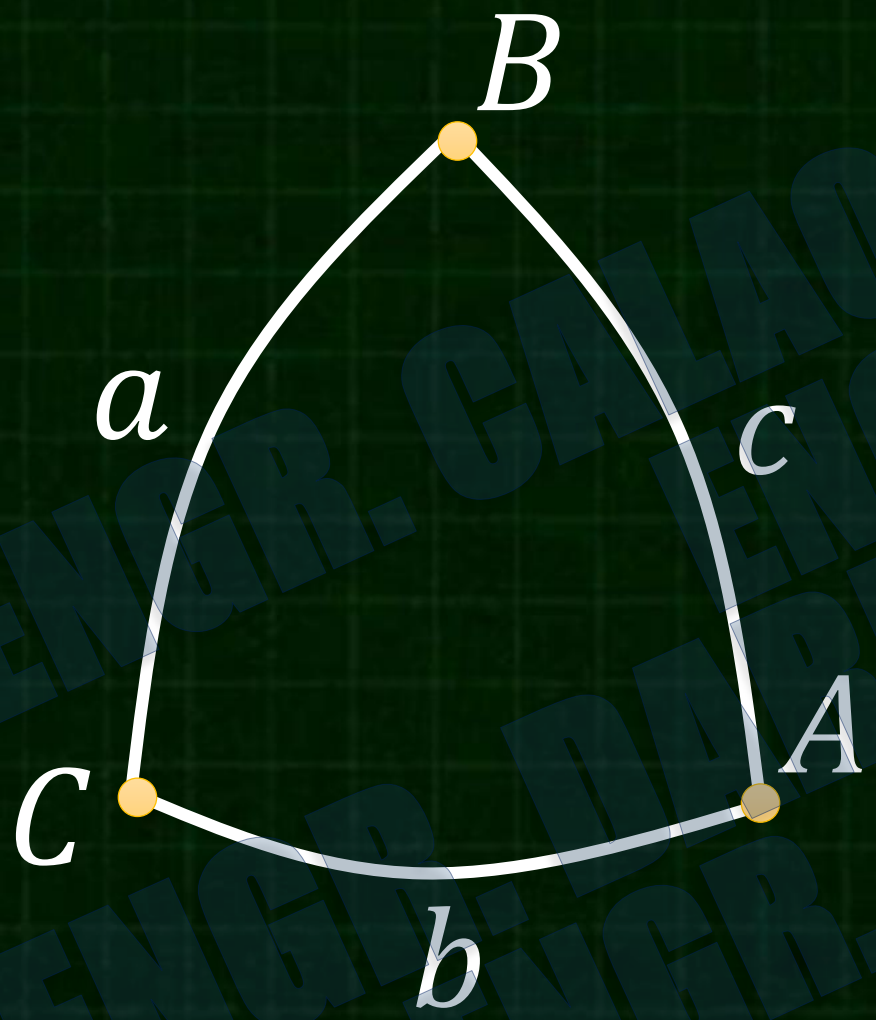
AREA OF SPHERICAL TRIANGLE



$$A = \frac{\pi R^2 E}{180}$$

WHERE:

E = SPHERICAL EXCESS IN DEGREES

$$E = (A + B + C) - 180$$

SPHERICAL DEFECT, d

$$d = 360 - (a + b + c)$$

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NOTES TO REMEMBER:

8 letter word

1NM (NAUTICAL MILE) = 6080FEET

MARINE AND AIR NAVIGATION

6 letter word

1NM = 1 MINUTE OF THE GREAT CIRCLE AREA ON THE SURFACE OF THE EARTH

5280

1 STATUTE MILE = 5280FEET

LAND DISTANCE MEASUREMENT

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PAST CE BOARD EXAM MAY 1997:

A SPHERICAL TRIANGLE ABC HAS AN ANGLE C = 90^0 AND SIDES a = 50^0 AND c = 80^0. FIND THE VALUE OF "b" IN DEGREES

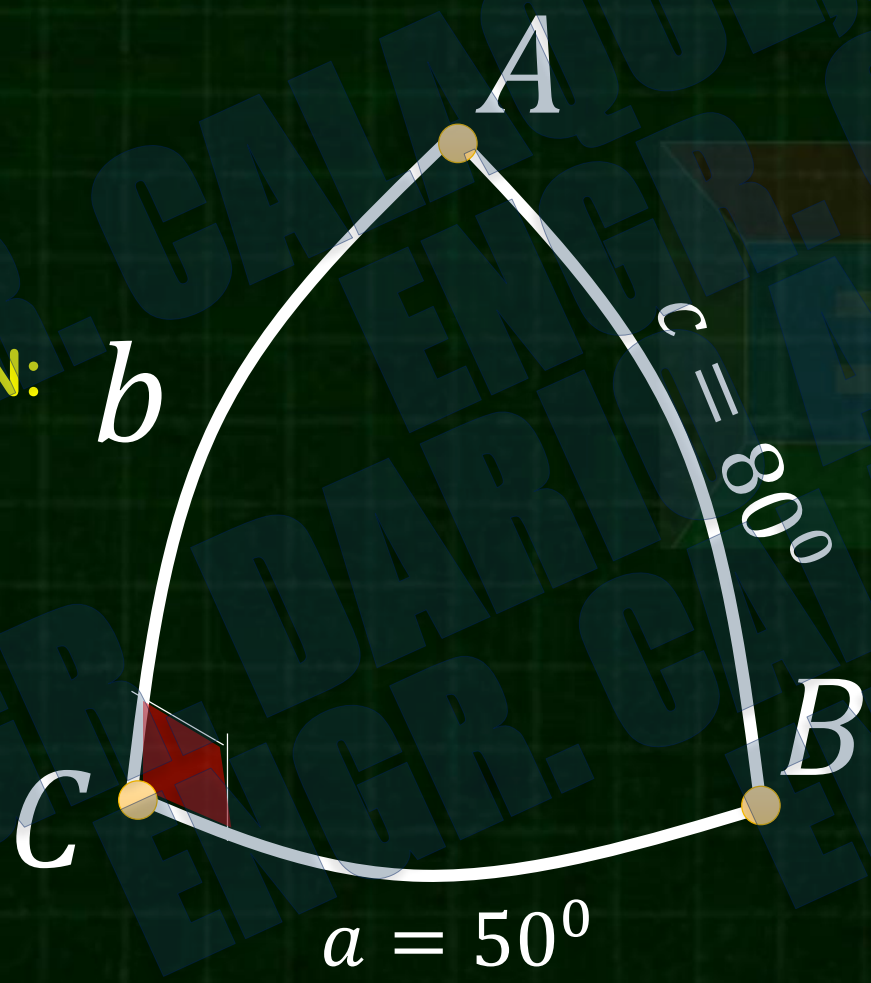
A) 73.22

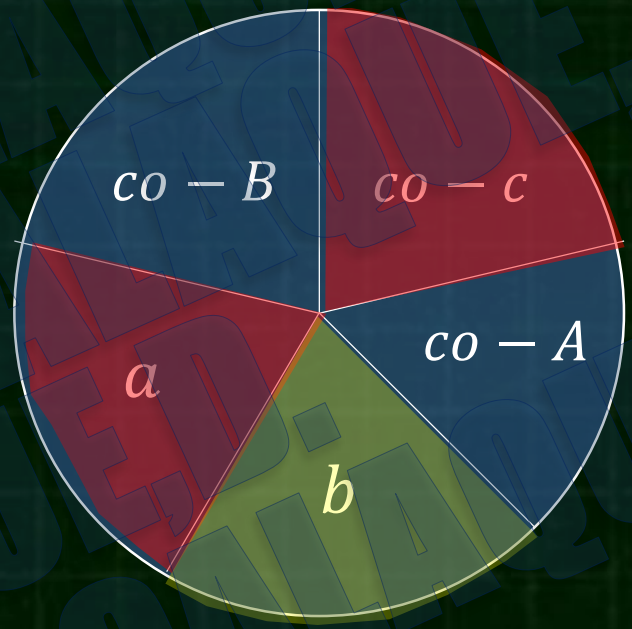
B) 74.33

C) 75.44

D) 76.55

SOLUTION:





$$Rule\ 2: CO - OP - RULE$$

$$\sin(co - c) = \cos(a)\cos(b)$$

$$\cos c = \cos(a)\cos(b)$$

$$\cos 80^0 = \cos(50^0)\cos(b)$$

$$\boxed{b = 74.33^0}$$

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PAST CE BOARD EXAM:

DETERMINE THE SPHERICAL EXCESS OF THE SPHERICAL TRIANGLE ABC GIVEN a = 56^0, b = 65^0 AND c = 78^0

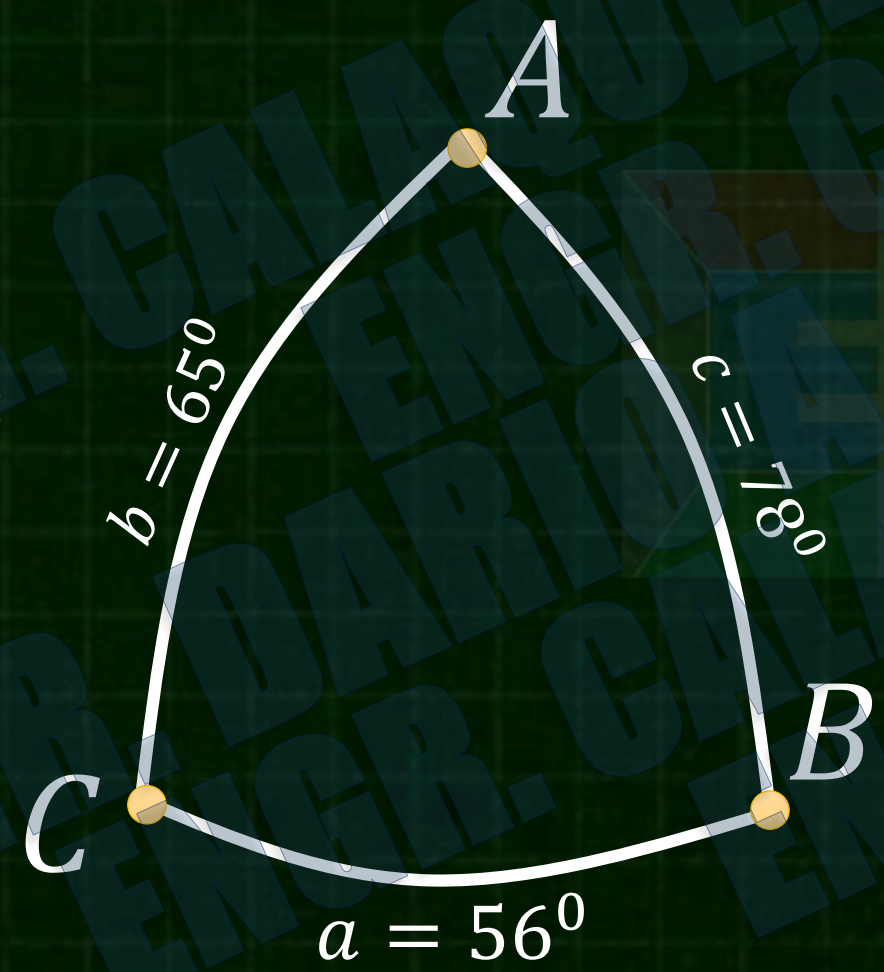
a) $33^0 33'$

b) $68^0 37'$

c) $91^0 57'$

d) $98^0 45'$

SOLUTION:



COSINE LAW (FOR THE SIDES):

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$
$$\cos b = \cos a \cos c + \sin a \sin c \cos B$$
$$\cos c = \cos a \cos b + \sin a \sin b \cos C$$

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$
$$\cos 56^0 = \cos 65^0 \cos 78^0 + \sin 65^0 \sin 78^0 \cos A$$
$$A = 57^0 52' 54.27''$$

$$\cos b = \cos a \cos c + \sin a \sin c \cos B$$
$$\cos 65^0 = \cos 56^0 \cos 78^0 + \sin 56^0 \sin 78^0 \cos B$$
$$B = 67^0 48' 11.96''$$

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PAST CE BOARD EXAM:

DETERMINE THE SPHERICAL EXCESS OF THE SPHERICAL TRIANGLE ABC GIVEN a = 56^0, b = 65^0 AND c = 78^0

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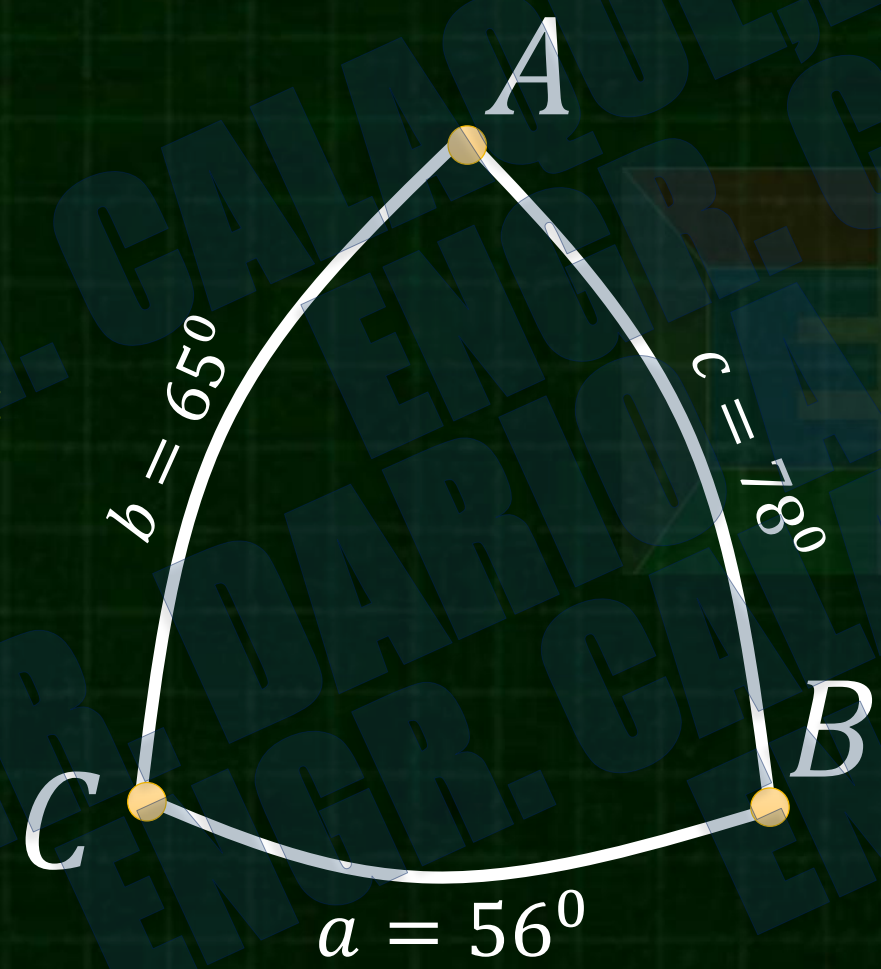

SINE LAW:

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

$$\frac{\sin 56^0}{\sin 57^0 52' 54.27''} = \frac{\sin 65^0}{\sin B}$$

$$B = 67^0 48' 11.97''$$

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PAST CE BOARD EXAM:

DETERMINE THE SPHERICAL EXCESS OF THE SPHERICAL TRIANGLE ABC GIVEN a = 56^0, b = 65^0 AND c = 78^0

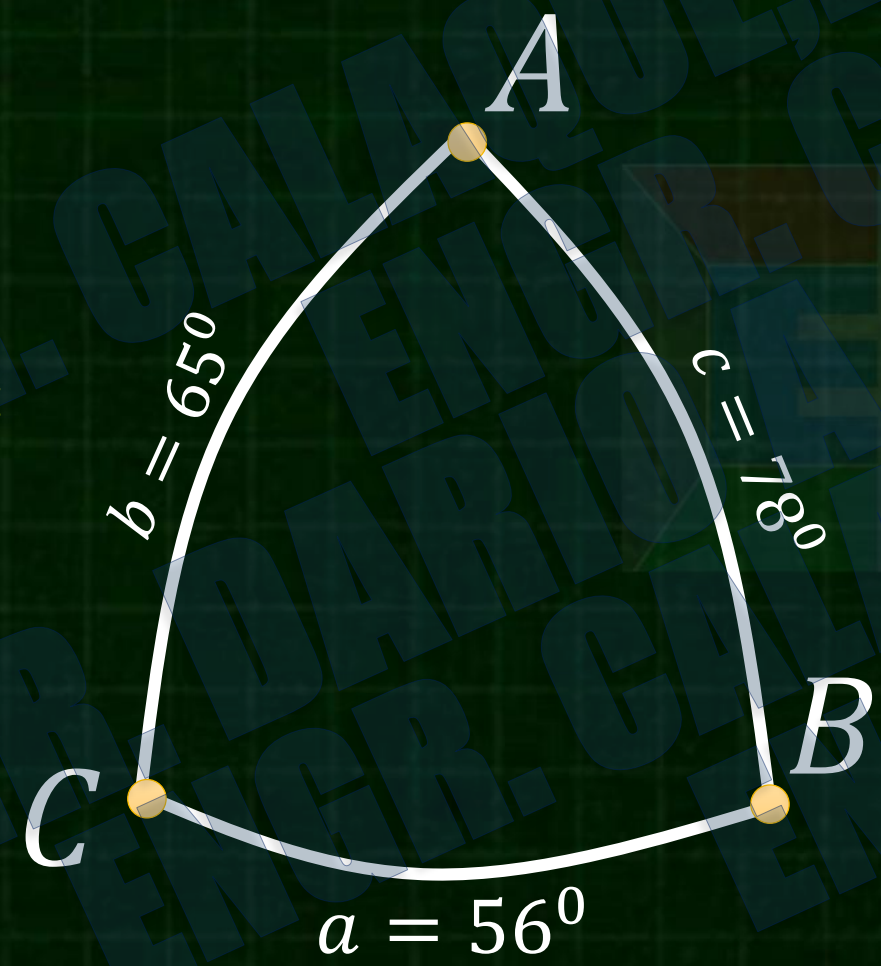
a) $33^0 33'$

b) $68^0 37'$

c) $91^0 57'$

d) $98^0 45'$

SOLUTION:



SINE LAW:

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

$$\frac{\sin 56^0}{\sin 57^0 52' 54.27''} = \frac{\sin 78}{\sin C}$$

$$C = 87^0 49' 58.13''$$

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PAST CE BOARD EXAM:

DETERMINE THE SPHERICAL EXCESS OF THE SPHERICAL TRIANGLE ABC GIVEN a = 56^0, b = 65^0 AND c = 78^0

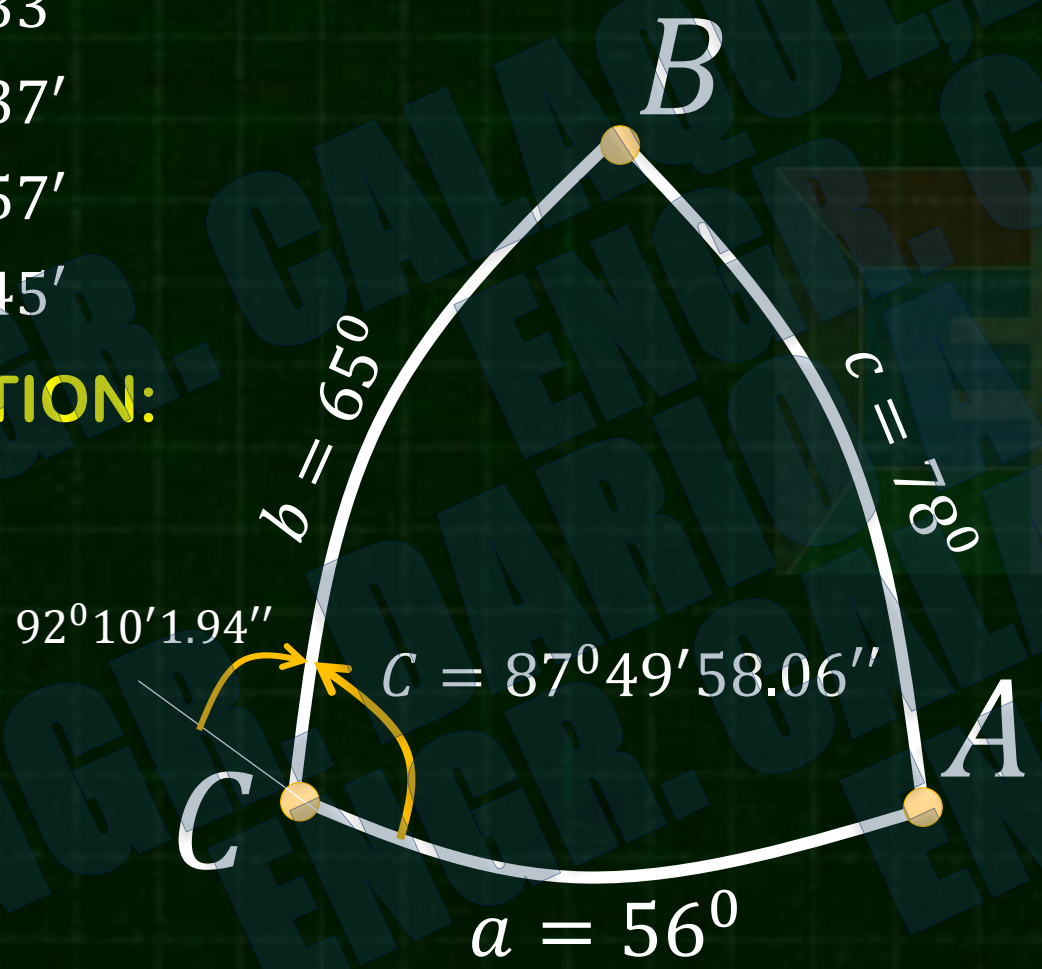
a) $33^0 33'$

b) $68^0 37'$

c) $91^0 57'$

d) $98^0 45'$

SOLUTION:



COSINE LAW (FOR THE SIDES):

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$
$$\cos b = \cos a \cos c + \sin a \sin c \cos B$$
$$\cos c = \cos a \cos b + \sin a \sin b \cos C$$

$$\cos c = \cos a \cos b + \sin a \sin b \cos C$$
$$\cos 78^0 = \cos 56^0 \cos 65^0 + \sin 56^0 \sin 65^0 \cos C$$
$$\cos C = -0.037815854$$
$$C = 92^0 10' 1.94''$$

$$C = 180^0 - (92^0 10' 1.94'')$$
$$C = 87^0 49' 58.06''$$

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PAST CE BOARD EXAM:

DETERMINE THE SPHERICAL EXCESS OF THE SPHERICAL TRIANGLE ABC GIVEN a = 56^0, b = 65^0 AND c = 78^0

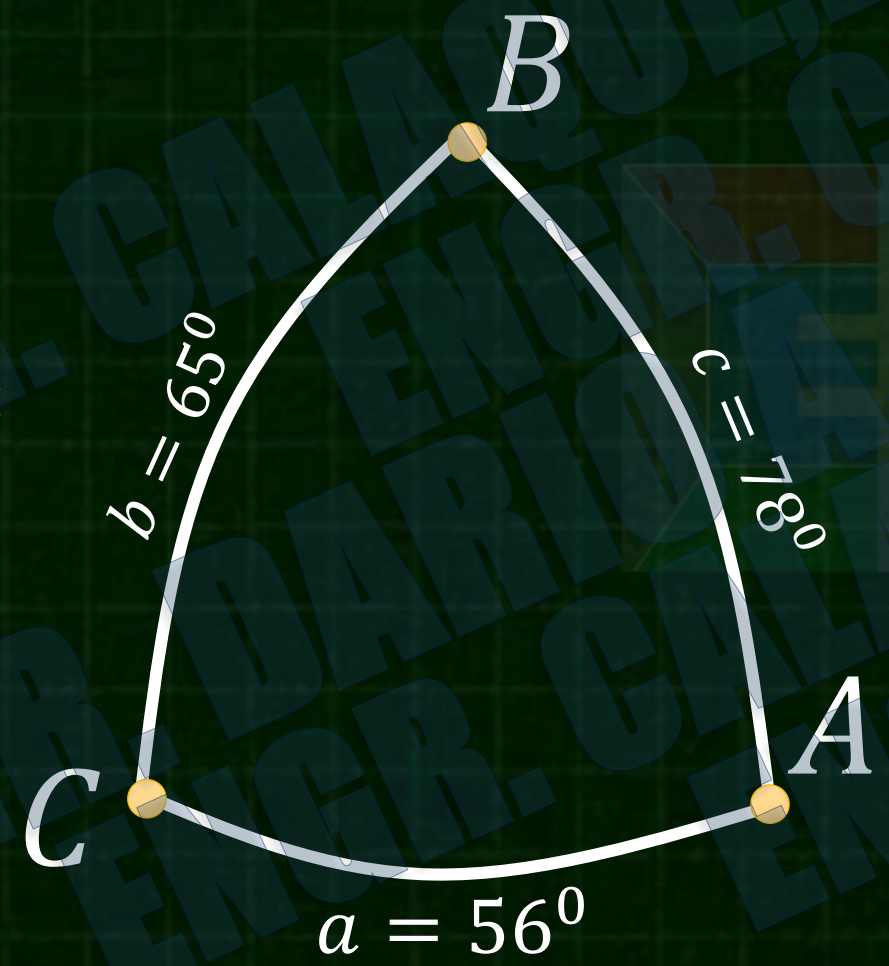
a) $33^0 33'$

b) $68^0 37'$

c) $91^0 57'$

d) $98^0 45'$

SOLUTION:



SPHERICAL EXCESS, E:

$A = 57^0 52' 54.27''$

$B = 67^0 48' 11.96''$

$C = 87^0 49' 58.13''$

$$E = (A + B + C) - 180^0$$

$$\boxed{E = 33^0 31' 4.36''}$$

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PAST CE BOARD EXAM:

A SPHERICAL TRIANGLE HAS AN AREA OF 327.25 SQ.KM. WHAT IS THE RADIUS OF THE SPHERE IF ITS SPHERICAL EXCESS IS 30DEGREES?

A) 20KM

B) 22KM

C) 25KM

D) 28KM

SOLUTION:

$$A = \frac{\pi R^2 E}{180}$$

$$327.25 = \frac{\pi R^2 (30^0)}{180}$$

$$R = 25km$$

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TEST YOURSELF:

For a spherical triangle whose parts are a = 73^0, b = 62^0 and C = 90^0. Determine the angle A

A. 74.89^0

B. 82.11^0

C. 63.05^0

D. 26.95^0

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TEST YOURSELF:

For a spherical triangle whose parts are a = 73^0, b = 62^0 and C = 90^0. Determine the angle B

A. 74.89^0

B. 82.11^0

C. 63.05^0

D. 26.95^0

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TEST YOURSELF:

For a spherical triangle whose parts are a = 73^0, b = 62^0 and C = 90^0. Determine the side, c.

A. 74.89^0

B. 82.11^0

C. 63.05^0

D. 26.95^0

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TEST YOURSELF:

Find the area of a spherical triangle of whose angles are 123^0, 84^0, and 73^0. The radius of the sphere is 30m.

A. $1863.3 sq.m$

B. $1570.8 sq.m$

C. $1958.6 sq.m$

D. $1480.2 sq.m$

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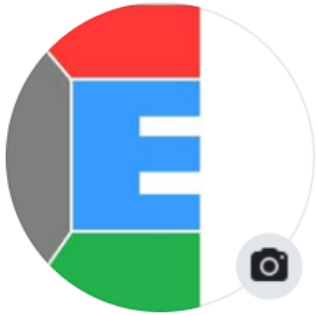

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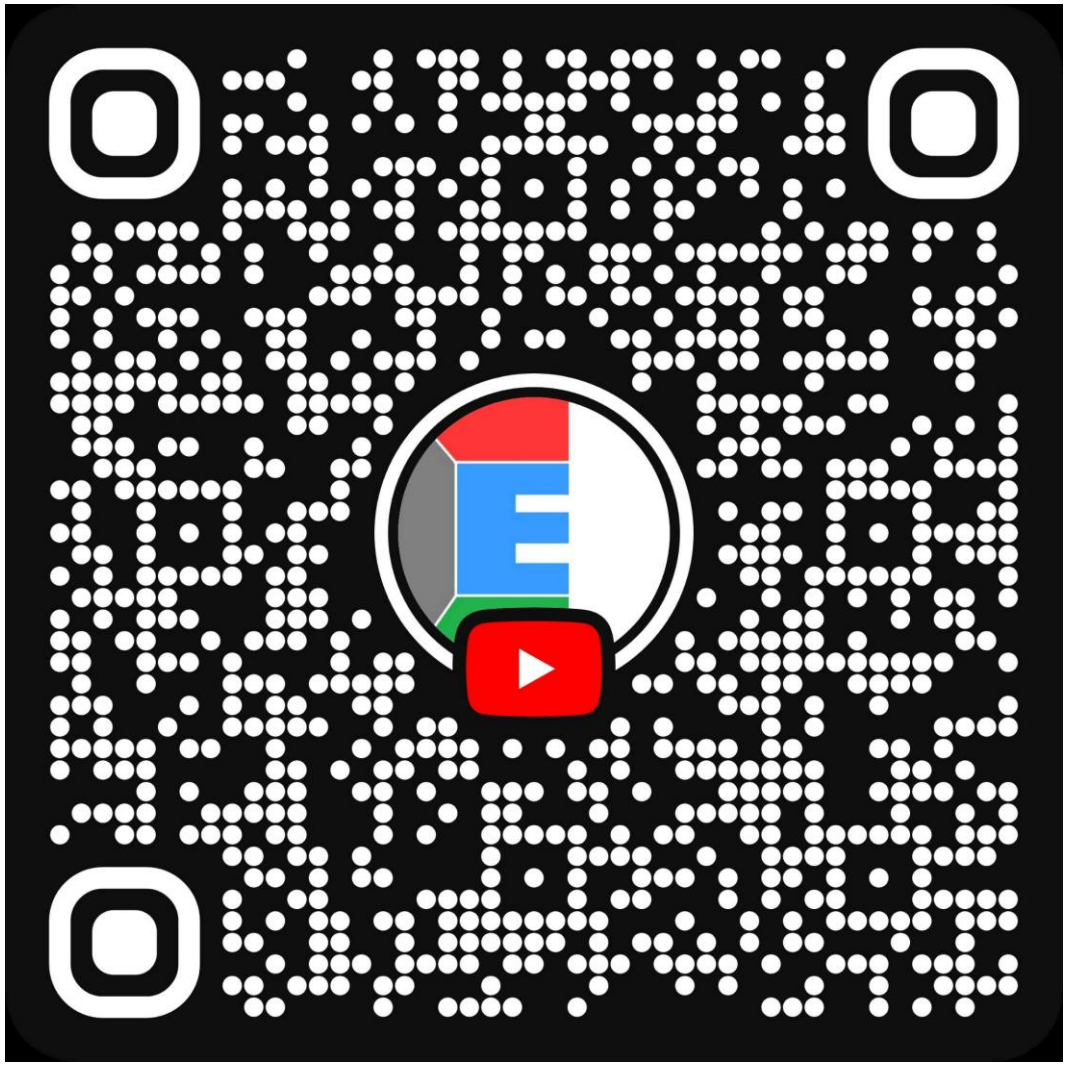

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